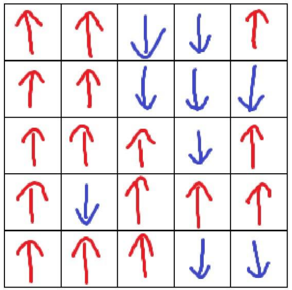
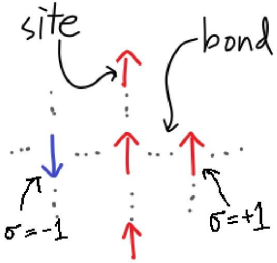
A course on the Ising model





Thursdays 9h45-11h45, Room 15-25.101 (Campus Pierre et Marie Curie)

Start date: 30 January 2025

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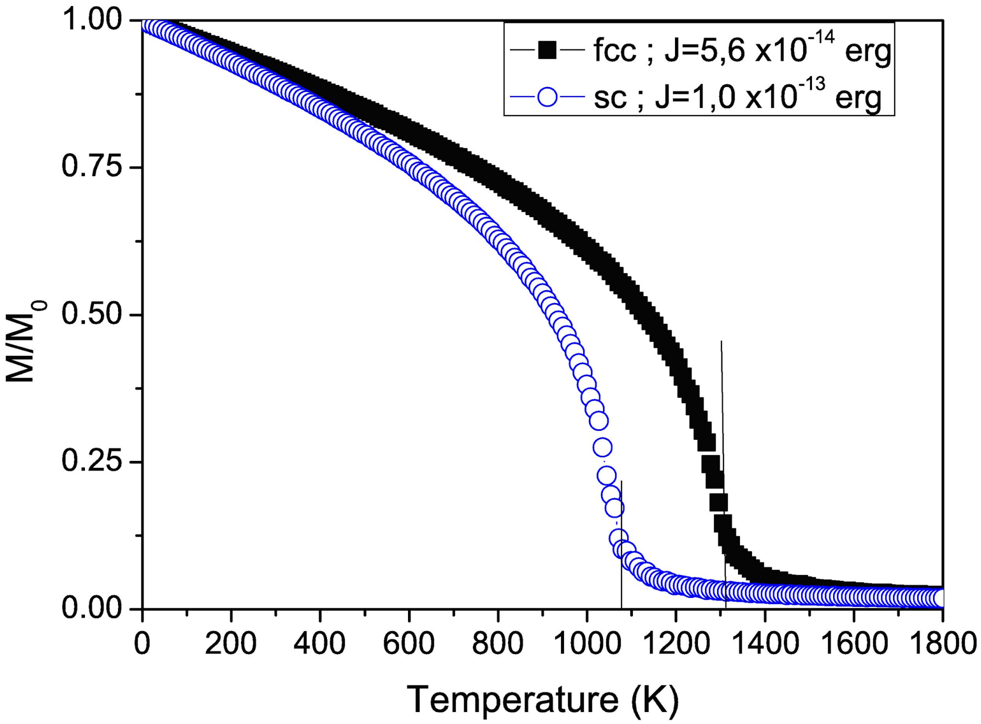
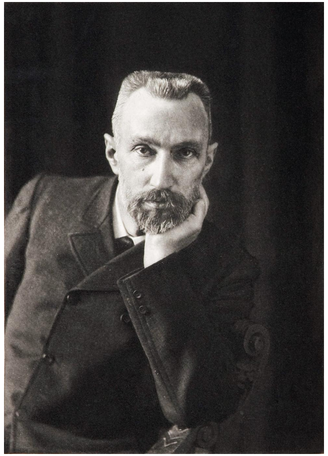
Pierre Curie observed that a magnet's strength depends on the temperature





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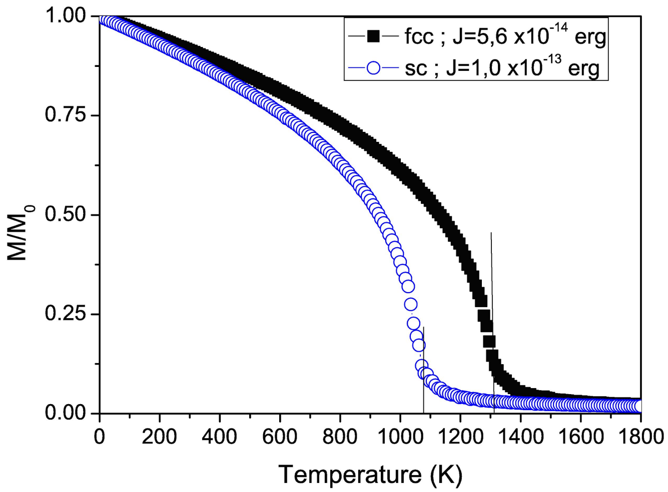

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- What happens at the **critical point**?
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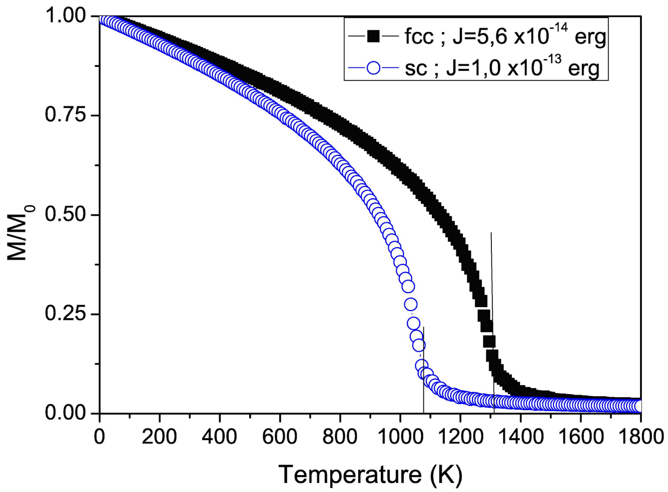

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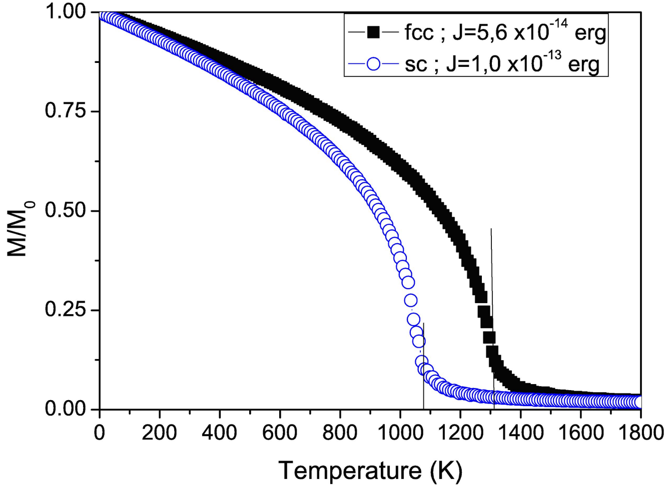

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Phase transition

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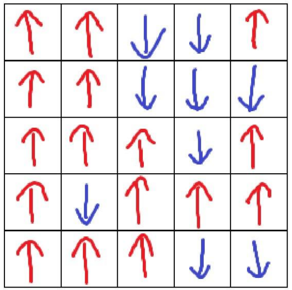

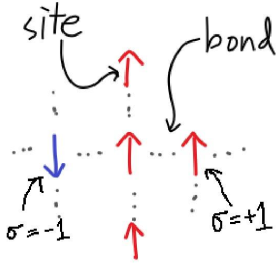

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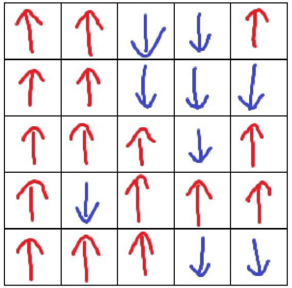

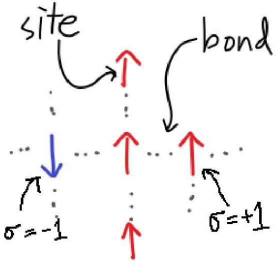

Ising solved the model in **one dimension** (one hundred years ago, in 1925). With the knowledge of today, this is straightforward because we have the theory of **Markov chains**.

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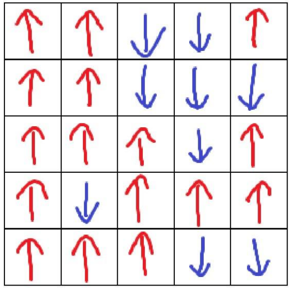
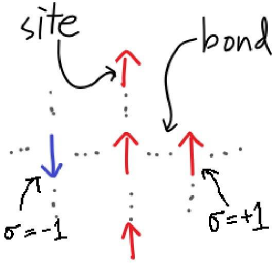




Ising: "The model has no phase transition!"

Ising solved the model in **one dimension** (one hundred years ago, in 1925). With the knowledge of today, this is straightforward because we have the theory of **Markov chains**.

Ising was right in 1D, but higher dimensions is much more complicated!

Complications arise because:

- 1D is straightforward because of Markov chains, but
- In higher dimensions, the geometry plays a huge role!

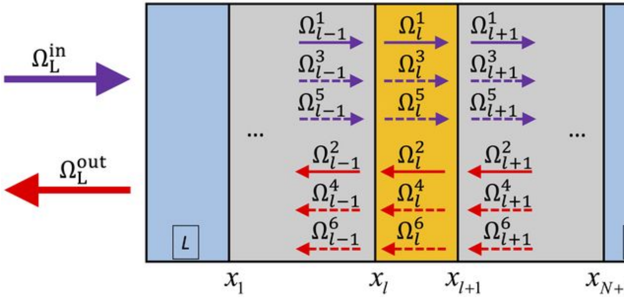
Example:

- Random walk is recurrent in 2D, but transient in 3D and higher D,
- Paths of random walks intersect in 3D, but not in 4D and higher D,

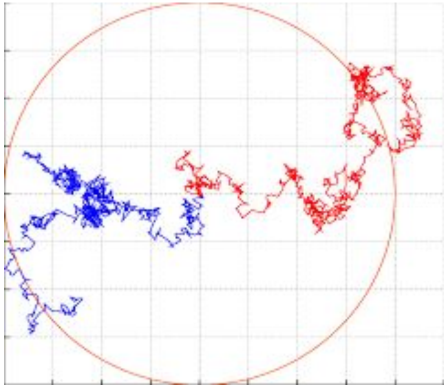
The Ising model:

- Behaves very differently in dimensions 1, 2, 3, 4, and 5+

The Ising model per dimension



- 1D: Trivial, Markov chains
- 2D: Almost everything is known (integrability, transfer matrix)
- 3D: Almost nothing is known
- 4D: Many things are known (random walks behave well)
- 5D: Almost everything is known (random walks behave very well)



Objectives of the course:

- Overview of **statistical mechanics (goals, ideas, methods)**
- Know the **key tools** and how to apply them
- Have an **historical overview** of the developments

Mathematically:

- Existence & uniqueness of the phase transition (1936 Peierls argument)
- Sharpness of the phase transition
- Continuity of the phase transition (2022 Fields medal, Duminil-Copin)

(Pre)requisites:

- Basic understanding of probability theory and graph theory
- The course will be reasonably demanding:
  - Some time to reflect on the material in between lectures will be useful

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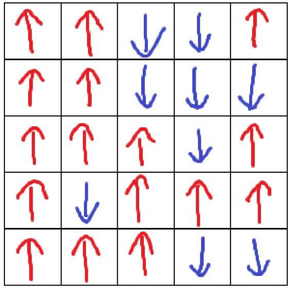

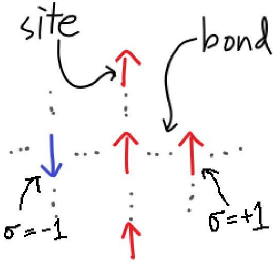

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